Computational Study on Soliton–like Pulses in the Nonlinear RLC Transmission Lines

Dalibor L. Sekulic* and Milos B. Zivanov*
* University of Novi Sad, Faculty of Technical Sciences / Department of Electronics, Novi Sad, Serbia
E-mails: dalsek@yahoo.com or dalsek@uns.ac.rs

Abstract - The nonlinear transmission line is a structure where short–duration pulses called electrical solitons can be created and propagated. In this paper, a method to study soliton–like pulses propagation along the nonlinear RLC transmission lines periodically loaded with voltage dependent capacitances is presented. Based on Kirchhoff's law taken in the continuum limit, exact soliton solutions of voltage equation are obtained by symbolic computation using an improved tanh–function method. It is shown that the shape of soliton can be controlled well by adjusting the parameters of the line. These solutions may have important applications in communication systems where solitons are used to codify data.

I. INTRODUCTION

Solitons, self–localized waves, are a special class of pulse–shaped waves that propagate without changing their shape in nonlinear dispersive media. A balancing mechanism between nonlinearity and dispersion is responsible for the appearance of solitary wave or pulse which propagates infinitely [1]. The first–reported soliton was a mono–pulse water wave in a narrow canal where the shallow water possessed both nonlinearity and dispersion. The propagation of modulated waves, such as bright or dark solitons, has been the subject of considerable interest for many years. The optical fiber is yet another example of a nonlinear dispersive medium where optical solitons are observed.

In electronics, nonlinear transmission lines (NLTLs) serve as nonlinear dispersive media where electrical solitons can propagate in the form of voltage waves [2]. A NLTL is a ladder network of repeating inductors and capacitors, where the inductors, the capacitors, or both are nonlinear in their response to current and voltage, respectively. The semiconductor devices such as Schottky diodes and heterostructure barrier varactors can also be incorporated into a soliton generating NLTL. These devices are used to generate oscillations with frequencies up to the terahertz range and pulses with subpicosecond durations [3]. Recently, it has been pointed out that a spatially modulated nonlinear dielectric line can be used to generate solitons at high powers and microwave frequencies. In this type of line, the dispersion comes from the periodicity of the line, while the nonlinearity comes from the dielectric material [4].

These lines are of interest because of their applications in several fields. For example, in the linear regime, the NLTLs can be used as phase shifters in phased antenna arrays [5], where time delay can be controlled by means of a DC bias applied to Schottky diodes acting as variable reactance. Under large signal conditions, NLTLs can serve as impulse compressors or frequency multipliers [6]. Recently, these lines have proven to be of great practical use in extremely wideband (frequencies from DC to 100 GHz) focusing and shaping of signals [7], which is usually a hard problem. Very recently, attempts have been made to study analytically and numerically nonlinear excitations using NLTLs, and it has been shown experimentally that solitons can exist in such systems [8].

The main aim of this paper is to present a method to computational study of solitary waves in nonlinear RLC transmission lines. A circuit model of a NLTL with N identical cells is given in Fig. 1. The series inductance is due to magnetic field effects, and the capacitance is due to electric field coupling between the lines. The losses in the transmission media are depicted by the series and the shunt resistors. These resistors represent the finite conductivity of the conductors and the dielectric insulator between the conductors, respectively. The resistors R_1 and R_2, accounting for the transmission line losses, the linear inductance L, and the voltage–dependent capacitance $C(V_n)$ are the circuit parameters. The resulting circuit is referred to as a distributed model of a nonlinear RLC transmission lines. A distributed model of the NLTL has been proposed to deal with wide–bandwidth signals.

Interestingly, in constructing proposed lossy nonlinear network, we were motivated with a very interesting example of nanobioelectronics problem. Namely, our group was recently modeled ionic currents along microtubules in the eukaryotic cells [9]. We analytically analyzed the possible use of microtubules as protein structure for building biomolecular nanoscale NLTLs in the context of the polyelectrolyte character of these cytoskeletal filaments [10].

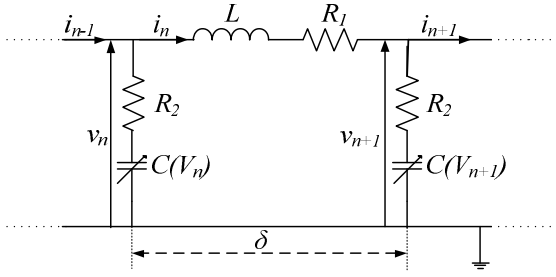

Figure 1. Schematic representation of one cell of the nonlinear transmission line.

The structure of the rest of paper is the following. In Section 2, we present the characteristic of the NLTL under consideration and we write down the voltage equations governing small–amplitude pulses in the transmission line shown in Fig. 1. We undertake the study of the propagation of solitons on the line in Section 3. By applying the symbolic computation using improved tanh–function method, we obtain exact analytical soliton–like solutions of the NLTL. Simulation results for verification of the obtained solution are given in Section 4. Section 5 summarizes results of this paper.

II. Model Description and Voltage Equation

We now consider a NLTL constructed by N elementary cells, where each cell, such as the nth one, contains a linear inductance L in the series branch, a nonlinear capacitor $C(V_b+V_n)$ in the shunt branch with V_n being the voltage across it and V_b a constant DC bias voltage, and the linear resistances R_1 and R_2, see Fig. 1. The reverse–biased diode or MOS varactor can serve as the nonlinear capacitor, and its differential capacitance is $C(V_b+V_n) = dq_n/dV_n$, where q_n denotes the stored charge in the nth capacitor, and it depends nonlinearly on the voltage V_n. For low voltages around the DC bias voltage V_b, the nonlinear capacitance can be approximated by [11]

$$C(V_b+V_n)=\frac{dq_n}{dV_n}=C_0\left(1+2\alpha V_n+3\beta V_n^2+...\right), \quad (1)$$

where C_0 is the linear capacitance of the capacitor, and α and β designate the nonlinear coefficients of the capacitor.

Equation (1) can be the second order curve fitting for the diode characteristics or the varactor characteristics according to the sign of nonlinear coefficients α and β. For the diode the sign of coefficient α is negative and the sign of coefficient β is positive and vice versa for the MOS varactor. In this study, we only consider the case when the perturbation voltage is small in comparison with the equilibrium voltage V_b, therefore, we neglect higher–order terms in (1), keeping only the first two terms of the expansion.

By applying the Kirchhoff current law at node n whose voltage with respect to the ground is v_n, and applying the Kirchhoff voltage law across the two inductors connected to this node, we obtain:

$$i_{n-1}-i_n=\frac{dq_n}{dt}, \quad (2)$$

$$v_n-v_{n+1}=L\frac{di_n}{dt}+R_1 i_n. \quad (3)$$

Similarly, if the voltage across the capacitor is V_b+V_n, where V_b is the bias voltage of the capacitor, we have:

$$v_n=R_2(i_{n-1}-i_n)+V_b+V_n. \quad (4)$$

From (3) we have:

$$\begin{aligned} L\frac{di_{n-1}}{dt}&=v_{n-1}-v_n-R_1 i_{n-1},\\ L\frac{di_n}{dt}&=v_n-v_{n+1}-R_1 i_n. \end{aligned} \quad (5)$$

From (2) we have

$$L\frac{d^2q_n}{dt^2}=L\frac{di_{n-1}}{dt}-L\frac{di_n}{dt}, \quad (6)$$

and including (5) in (6), we get:

$$L\frac{d^2q_n}{dt^2}=v_{n-1}-2v_n+v_{n+1}+R_1(i_n-i_{n-1}). \quad (7)$$

Replacing the expressions for the voltages v_{n-1}, v_n and v_{n+1} from (4) to (7), we obtain that voltages of the adjacent nodes on this lossy NLTL are related via partial differential equation (PDE) as follows

$$\begin{aligned} &L\frac{d^2}{dt^2}\left[C_0\left(V_n+\alpha V_n^2\right)\right]+R_1C_0\frac{d}{dt}\left(V_n+\alpha V_n^2\right)-\\ &-R_2C_0\left\{\frac{d}{dt}\left(V_{n-1}+\alpha V_{n-1}^2\right)-2\frac{d}{dt}\left(V_n+\alpha V_n^2\right)\right.\\ &\left.+\frac{d}{dt}\left(V_{n+1}+\alpha V_{n+1}^2\right)\right\}=V_{n-1}-2V_n+V_{n+1}. \end{aligned} \quad (8)$$

The right–hand side of (8) can be approximated with partial derivatives with respect to distance x, from the beginning of the line, assuming that the spacing between two adjacent sections is δ (i.e., $x_n = n\delta$). Let voltag $V(x, t)$ be a continuous function of the variables x and t so that $V(n, t) = V_n(t)$ and $V_{n\pm1}(t) = V(x\pm\delta, t)$. An approximate continuous PDE can be obtained by using the Taylor expansions

$$V_{n\pm1}=V(x\pm\delta,t)=V\pm\delta\frac{\partial V}{\partial x}+\frac{\delta^2}{2!}\frac{\partial^2V}{\partial x^2}\pm\frac{\delta^3}{3!}\frac{\partial^3V}{\partial x^3}+.., \quad (9)$$

to evaluate the right–hand side of (8), i.e.,

$$V_{n-1}-2V_n+V_{n+1}=\delta^2\frac{\partial^2V}{\partial x^2}+\frac{\delta^4}{12}\frac{\partial^4V}{\partial x^4}. \quad (10)$$

By eliminating terms of higher order than δ^4 as negligible small and considering that the time variations of local voltage V are small compared to the constant background voltage V_b, we could safely assume that time derivative is of the order of small parameter ε as well as the nonlinear voltage terms αV^2 are of the order of ε^2. Thus, we obtain the following nonlinear PDE for the perturbed voltage V:

$$R_2 C_0 \delta^2 \frac{\partial}{\partial t}\left(\frac{\partial^2 V}{\partial x^2}\right) + \delta^2 \frac{\partial^2 V}{\partial x^2} - LC_0 \frac{\partial^2 V}{\partial t^2} + 2\alpha R_1 C_0 V \frac{\partial V}{\partial t} - R_1 C_0 \frac{\partial V}{\partial t} = 0. \quad (11)$$

III. SOLITON–LIKE PULSES IN NLTL

In this study, the improved tanh–function method (ITM) is applied in order to find the analytical solution of the NLTL [12]. The first step in this method is to introduce the voltage in the form of the travelling wave

$$V(x,t) = V(\xi); \quad \xi = x - \mathrm{v}_0 t, \quad (12)$$

where v_0 is undetermined parameter that represents the velocity of propagation. A propagating mode solution of (11) can be obtained by substituting (12) into the PDE (11). Then voltage equation (11) becomes the ordinary differential equation (ODE) of the third order

$$\frac{R_2}{R_1}\delta^2 \mathrm{v}_0 \frac{d^3 V}{d\xi^3} + \frac{L}{R_1}\left(\mathrm{v}_0^2 - \delta^2 \omega_\mathrm{c}^2\right)\frac{d^2 V}{d\xi^2} + 2\alpha \mathrm{v}_0 V \frac{dV}{d\xi} - \mathrm{v}_0 \frac{dV}{d\xi} = 0. \quad (13)$$

Here the parameter ω_c, meaning the cutoff angular frequency, is defined by $\omega_\mathrm{c} = (LC_0)^{-1/2}$ and the maximum velocity of the propagation of waves is $\mathrm{v}_{0\max} = \delta\omega_\mathrm{c}$. After first integration of ODE (13), we obtain

$$A\frac{d^2 V}{d\xi^2} + B\frac{dV}{d\xi} - \mathrm{v}_0 V + DV^2 = \mathrm{c}_1, \quad (14)$$

where c_1 is an arbitrary constant, whose value depends on the initial conditions. Requiring that all functions $V(\xi)$, $dV(\xi)/d\xi$ and $dV^2(\xi)/d\xi^2$ tend to zero as $\xi \to \infty$, the integration constant c_1 is set to be zero. The parameters of ODE (14) read as follows:

$$A = \frac{R_2}{R_1}\delta^2 \mathrm{v}_0, \quad B = \frac{L}{R_1}\left(\mathrm{v}_0^2 - \delta^2 \omega_\mathrm{c}^2\right), \quad D = 2\alpha \mathrm{v}_0. \quad (15)$$

It is observed form (13) that the sign of the dispersion coefficient, A, is always positive, but the sign of the nonlinearity coefficient, D, depends on the nonlinear characteristics of the capacitance. Therefore, the sign of the product AD can be positive or negative, which leads to appearance of dark soliton ($AD < 0$) or bright soliton ($AD > 0$). The parameter B is always positive and involves the effect of dissipation.

We now introduce the new independent variable $Y = \tanh\xi$ and $W(Y) = V(\xi)$ [13], as a polynomial series, due to which ODE (14) transforms to the shape

$$A\left(1-Y^2\right)^2 \frac{d^2 W}{dY^2} + \left\{B\left(1-Y^2\right) - 2AY\left(1-Y^2\right)\right\}\frac{dW}{dY} - \mathrm{v}_0 W + DW^2 = 0. \quad (16)$$

We compare the derivative term of highest–order with the highest order nonlinear term. So, balancing the order of W'' with the order of W^2 in (16), we obtain $m + 2 = 2m \to m = 2$. So the solution takes the form

$$W(Y) = \sum_{i=0}^{m=2} a_i Y^i = a_0 + a_1 Y + a_2 Y^2, \quad (17)$$

where parameters a_0, a_1, and a_2 are to be determined. Inserting (17) into (16), we get a system of algebraic equations:

$$\begin{aligned}
Y^0: &\quad 2a_2 A + a_1 B + a_0^2 D - \mathrm{v}_0 a_0 = 0,\\
Y^1: &\quad -2a_1 A + 2a_2 B + 2a_0 a_1 D - \mathrm{v}_0 a_1 = 0,\\
Y^2: &\quad -8a_2 A - a_1 B + a_1^2 D + 2a_0 a_2 D - \mathrm{v}_0 a_2 = 0,\\
Y^3: &\quad 2a_1 A - 2a_2 B + 2a_1 a_2 D = 0,\\
Y^4: &\quad 6a_2 A + a_2^2 D = 0.
\end{aligned} \quad (18)$$

Solving the above system of algebraic equations with the aid of MATLAB, we obtain the following calculated values of the parameters:

$$a_0 = \frac{1}{D}\left(6A + \frac{1}{2}\mathrm{v}_0\right); \quad a_1 = \frac{6}{5}\frac{B}{D}; \quad a_2 = -6\frac{A}{D}. \quad (19)$$

together with a relation between coefficients B and A as $B^2 = 100A^2$. Finally, using the determined parameters, the exact soliton solution of the voltage equation (11) reads explicitly

$$V(x,t) = \frac{1}{D}\left(6A + \frac{1}{2}\mathrm{v}_0\right) + \frac{6}{5}\frac{B}{D}\tanh\left(x - \mathrm{v}_0 t\right) - 6\frac{A}{D}\tanh^2\left(x - \mathrm{v}_0 t\right). \quad (20)$$

It represents a particular combination of a bell–shaped solitary wave, third term of (20), with a kink–like shock–wave, second term. Here, the solitary wave velocity v_0 is given by the following implicit expression

$$\mathrm{v}_0^2 = 4\left(24A^2 + \frac{6}{5}B^2\right). \quad (21)$$

IV. NUMERICAL SIMULATION AND RESULTS

In order to verify our analytical prediction based on (20), we have also analyzed the NLTL using well–known methods of circuit analysis. To this end, the circuit in Fig. 1 is connected to a source with 50 Ω resistance and a load

Figure 2. The 3-D plot of bright soliton at different positions, x, obtained from simulation of the NLTL.

with 50 Ω resistance. The number of cells between the source and the load are assumed to be 40. The NLTL is then analyzed using a standard time–domain technique for nonlinear circuits [14]. The method uses finite–difference applied to the differential equations governing the voltages and current of the circuit.

In the simulation, we first consider the NLTL with a varactor, the linear inductance L = 2.5 nH, and the linear resistances R_1 = 0.1 Ω, and R_2 = 1 Ω. The varactor's parameters are linear capacitance C_0 = 1 pF, and nonlinearity factor α = 0.2 V^{-1}. Because of the positive sign of the product AD, as predicted by the analytical solution, a bright soliton will be formed along the transmission line. The NLTL is excited by the input voltage given by [11]:

$$V_{in}(t) = V_m \operatorname{sech}\left(\frac{t-\tau_0}{T_0}\right)\cos(2\pi f t), \qquad (22)$$

where the amplitude V_m = 1 V, the parameters τ_0 = 0.3 ns, and T_0 = 1 ns. The carrier wave frequency is chosen to be f = 2.5 GHz in passband of the transmission line, below ω_c.

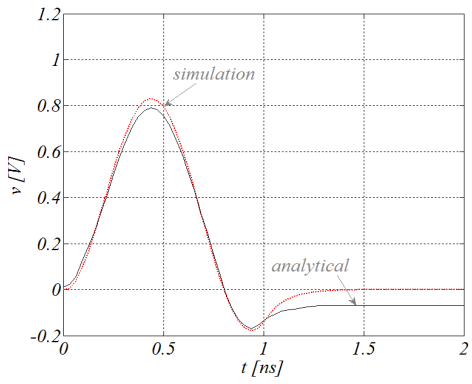

Figure 3. Comparison between the waveform envelopes obtained from the analytical solution and the simulation at x = 0.1 m.

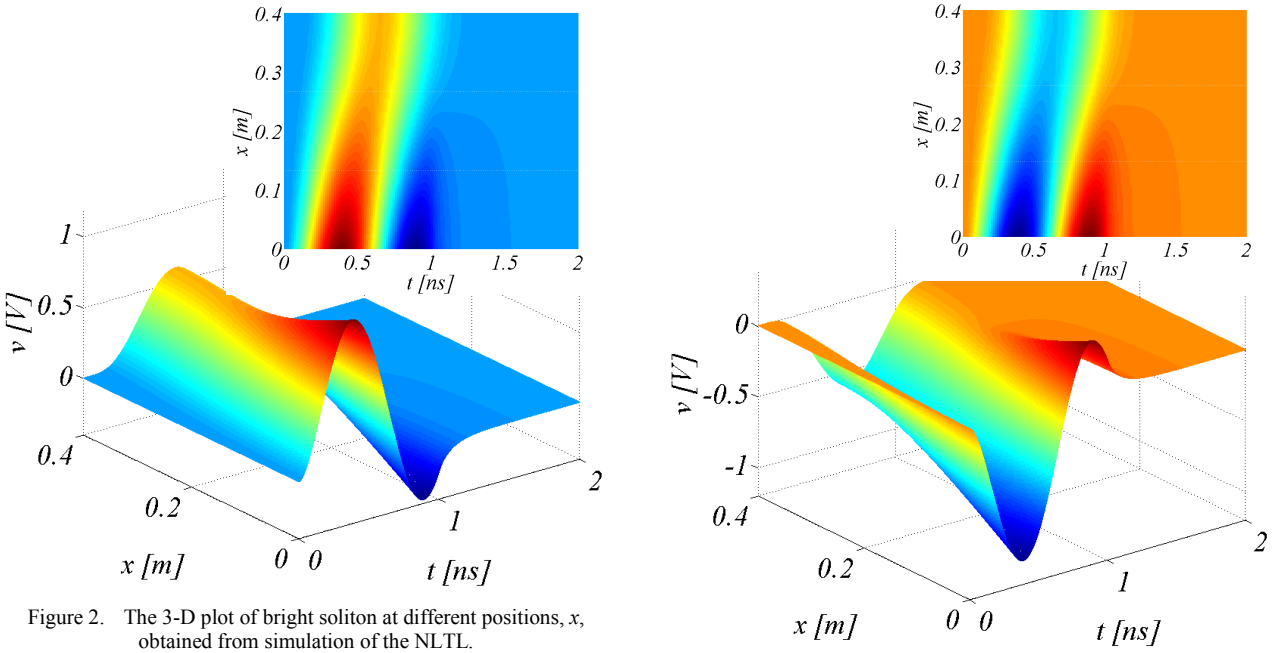

Figure 4. The 3-D plot of dark soliton at different positions, x, obtained from simulation of the NLTL.

Propagation of this pulse along the transmission line is simulated and the obtained envelope waveform at different positions, x, is depicted in Fig. 2. As seen in this figure, the soliton–like pulse preserves its shape along the propagation path. According to the analytical calculations, this circuit because of its positive nonlinearity cannot support dark solutions. As observed in Fig. 3, it presents a good agreement between the analytical and simulated results at x = 0.1 m.

Now, we consider the NLTL with a reverse–biased diode and the same values for the inductance and resistances. The diode's parameters are linear capacitance C_0 = 1 pF, and nonlinearity factor α = – 0.25 V^{-1}. In this case, the sign of the product AD is negative. Therefore, if this circuit is excited by a dark initial signal, a dark soliton will be generated. The envelope waveform obtained from the finite–difference method at different positions, x, is depicted in Fig. 4. As observed in Fig. 5, we get a good agreement between the analytical and simulated results. According to the analytical calculations, this circuit because of its negative nonlinearity cannot support bright solutions.

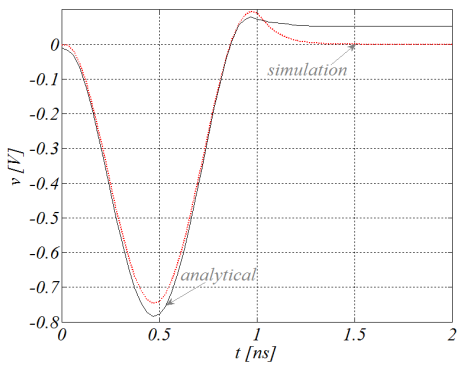

Figure 5. Comparison between the waveform envelopes obtained from the analytical solution and the simulation at x = 0.15 m.

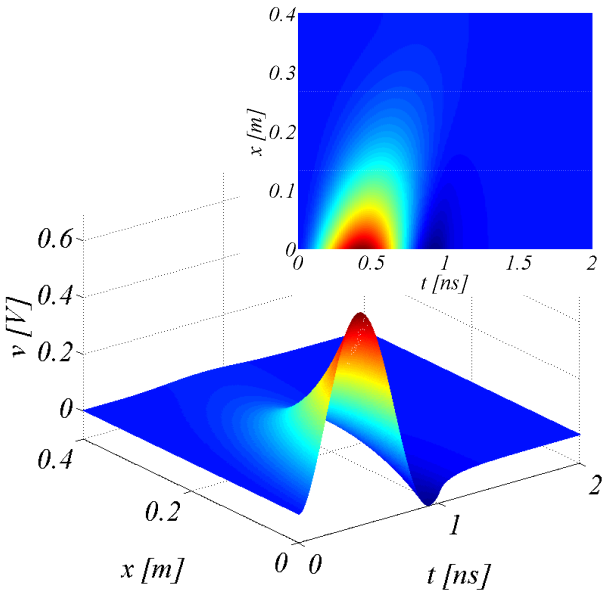

Figure 6. The influnce of dissipation on waveform of the envelope soliton.

Finally, let us consider the propagation of soliton–like waves in the case of larger values of resistance than in the first two cases. In this case, we consider the NLTL with a varactor, inductance, and linear resistances $R_1 = 10\ \Omega$, and $R_2 = 5\ \Omega$. The varactor's parameters and linear inductance are the same as observed in the first case. In Fig. 6 is shown the influence of dissipation along the line for the envelope soliton. We can conclude that dissipation acts on the amplitude of the soliton but not on its profile. The amplitude of soliton–like pulsess decreases with the increase of the resistances line. Thus, our results suggest that the losses must be reduced in order to improve the propagation of solitons.

V. CONLUSION

In this paper, we analytical and numerical studied the propagation of soliton–like signals in a nonlinear RLC transmission line. Using Kirchhoff's laws has been shown that the voltages along the line are governed by a third–order nonlinear PDE. The coefficients of the obtained voltage equation show that the dispersion coefficient, A, is always positive, but the sign of the nonlinearity coefficient, D, is determined by the characteristics of the varactor or the reversed biased diode. Depending on the sign of the product AD, the proposed NLTL can generate either bright or dark solitons. Also, we found the soliton solution of circuit equation using ITF method. From the analytical solution is possible to choose the suitable parameters of NLTLs to satisfy the required application or to compare the analytical results with the experimental results. We have verified generation of bright and dark solitons using a circuit simulator based on a finite–difference method. A good agreement is observed between the analytical prediction and the simulation results. Our results suggest that the losses must be reduced in order to improve the propagation of soliton–like pulses, because the amplitude of soliton decreases with the increase of the resistances line.

ACKNOWLEDGMENT

The authors would like to thank the Ministry of Education and Science of Serbia, Grant III43008, for the financial support of this work.

REFERENCES

[1] A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, "The soliton: A new concept in applied science," Proceedings of the IEEE, vol. 61, pp. 1443–1483, October 1973.

[2] M. J. Rodwell, S. T. Allen, R. Y. Yu, M. G. Case, U. Bhattacharya, M. Reddy, E. Carman, M. Kamegawa, Y. Konishi, J. Pusl, and R. Pullela, "Active and nonlinear wave propagation devices in ultrafast electronics and optoelectronics," Proceedings of the IEEE, vol. 82, pp. 1037–1059, Jul 1994.

[3] M. J. W. Rodwell, M. Kamegawa, R. Yu, M. Case, E. Carman, and K. S. Giboney, "GaAs nonlinear transmission lines for picosecond pulse generation and millimeter-wave sampling," IEEE Transactions on Microwave Theory and Techniques, vol. 39, pp. 1194–1204, July 1991.

[4] H. Ikezi, J. S. deGressie, and J. Drake, "Soliton generation at 10MW level in the very high frequency band," Applied PhysicsLetters, vol. 58, pp. 986–987, April 1991.

[5] W. M. Zhang, R. P. Hsia, C. Liang, G. Song, C. W. Domier, and N. C. Luhman, "Novel low-loss delay line for broadband phased antenna array applications," IEEE Microwave Guided Wave Letters, vol. 6, pp. 395–397, November 1996.

[6] E. Lheurette, M. Fernandez, X. Melique, P. Mounaix, O. Vanbesien, and D. Lippens, "Non linear transmission line quintupler loaded by heterostructure barrier varactors," 29th European Microwave Conference, pp. 217–222, 1999.

[7] E. Afshari, and A. Hajimiri, "Nonlinear transmission lines for pulse shaping in silicon," IEEE Journal of Solid-State Circuits, vol. 40, pp. 744–752, March 2005.

[8] R. Stearrett, and L. Q. English, "Experimental generation of intrinsic localized modes in a discrete electrical transmission line," Journal of Physics D: Applied Physics, vol. 40, pp. 5394–5398, September 2007.

[9] D. L Sekulić, B. M Satarić, J. A. Tuszynski, and M. V. Satarić, "Nonlinear ionic pulses along microtubules," European Physical Journal E: Soft Matter, vol. 34, art. no. 49, May 2011.

[10] D. L Sekulić, and M. V. Satarić, "Protein-based nanobioelectronics transmission lines," 28th International Conference on Microelectronics–MIEL, May 2012, in press.

[11] K. Tse Ve Koon, J. Leon, P. Marquie, and P. Tchofo-Dinda, "Cutoff solitons and bistability of the discrete inductance-capacitance electrical line: Theory and experiments," Physical Review E, vol. 75, pp. 066604.1–8, June 2007.

[12] S. Abdoulkary, A. Mohamadou, and T. Beda, "Exact traveling discrete kink-soliton solutions for the discrete nonlinear electrical transmission lines," Communications in Nonlinear Science and Numerical Simulation, vol. 16, pp. 3525–3532, September 2011.

[13] D. L. Sekulić, M. V. Satarić, and M. B. Živanov, "Symbolic computation of some new nonlinear partial differential equations of nanobiosciences using modified extended tanh-function method," Applied Mathematics and Computation, vol. 218, pp. 3499–3506, December 2011.

[14] D. L. Sekulić, M. V. Satarić, M. B. Živanov and J. S. Bajić, "Soliton-like pulses along electrical nonlinear transmission line," Electronics and Electrical Engineering, vol. 122, June 2012, in press.