Performance Analysis of Multiuser Diversity in Multiuser Two-Hop Amplify and Forward Cooperative Multi-Relay Wireless Networks

Mamoun F. Al-Mistarihi*, Ashraf Sharaqa* , Rami Mohaisen*, Omar Abu-Alnadi*, and Hani Abu-Seba*
* Jordan University of Science and Technology/ Electrical Engineering Department, Irbid, Jordan
mistarihi@just.edu.jo, elengashraf@yahoo.com, rami.fayez@hotmail.com, omar.abu-alnadi@hotmail.com, haniseba@gmail.com

Abstract - To adapt for the fading impairments and to provide a higher data rates wireless systems, a new shift in the perspectives came in the form of proposing a new evolving technology such as the multiuser diversity and the cooperative wireless networks which combines the features of the (MIMO) system without confronting the physical layer constraints by providing multiple copies of the transmitted signal from the source to the destination with the help of the relay node. In this paper, we present an analysis of a two hop cooperative multi-relay communication networks, derive a tight closed form expression of outage probability and symbol error probability (SEP) for the amplify-and-forward (AF) protocol with MUD.

Keywords: Amplify-and-forward (AF), cooperative communications, multiuser diversity (MUD), two hop cooperative multi-relay network (TCMRN).

I. INTRODUCTION

In communication systems designed to use a single signal path between transmitter and receiver have high probability to suffer from fading, such techniques are used to increase the signal to noise ratio at the receiver and reduce the SEP, the overall reliability can be improved by providing multiple copies of the transmitted signal to the receiver by using multiple relay nodes [1].

Cooperative networks diversity is available at the destination by using the intermediate relay nodes between the source and the destination which help the source in relaying it’s signal. Among the most commonly used schemes is the Amplify-and-forward (AF) which simply amplifies the received signal from the source and resends it to the destination. With this in mind, this is considered easy to implement as compared to the Decode and forward (DF) relaying scheme in which the relays decode the received signal, and re-transmit an encoded version to the receiver and hence the AF is much more preferable approach in wireless networks with simple relay units such as wireless sensor networks, besides that, the (AF) relying has low complexity compared to other techniques. This is based on the fact that the user will pass or go through a different fading conditions and at any given instance of time, there would be a user whose channel gain is the best among all the other users [2].

Many researches have been done on analyzing the performance of MUD in cooperative networks. In [3]–[6], the research of MUD can be considered as the extension to multiple-input–multiple-output antenna systems (MIMO) [7], [8]. The research of MUD in cooperative networks focus on network scenarios with a single source, one single destination, and M multiple relaying nodes in between, whereas, in [2] the authors extend the analysis to multi-user two-hop cooperative networks (TCRNs) with a single relaying node, one single destination, and multiple sources.

This paper extends the work of MUD in multiuser TCRNs to multiuser two-hop cooperative wireless networks with multiple relaying nodes network (TCMRNs) with multiple relays, one single destination, and multiple users (sources) and we derive tight closed form formulas of outage probability and symbol error probability (SEP) for the amplify-and-forward (AF) scheme with MUD.

The major contributions of this paper are summarized as follows:

1. A tight closed form expression of outage probability is derived for TCMRNs, which shows that the system outage probability will be reduced by a factor of $\bar{\gamma}^{2K}$ where K being the number of the available users.
2. A closed form SEP expression is obtained; from this formula, we show that diversity order of $2K$ is achieved for AF.

This paper is further presented as follows; Section II provides the system model. Section III provides the performance analysis and derived tight closed form formulas of outage probability and SEP. Results and discussion are presented in section IV. We concluded this paper in section V.

II. SYSTEM MODEL

Consider a multiuser TCMRN system with one destination, K-users and M-relays as illustrated in Fig. 1. Each user can communicate with the destination directly or via the relays. In this paper, we adopt the AF scheme

[9] and that the orthogonal channel access method being used in the proposed system is the time-division multiple-access (TDMA). For MUD, at the given time instant, one "best" user among all the available K users is elected to transmit the signal. At the destination the channel state information (CSI) is available and can properly be obtained by the virtue of using the feedback channels between the destination and the relay nodes for MUD-based scheduling.

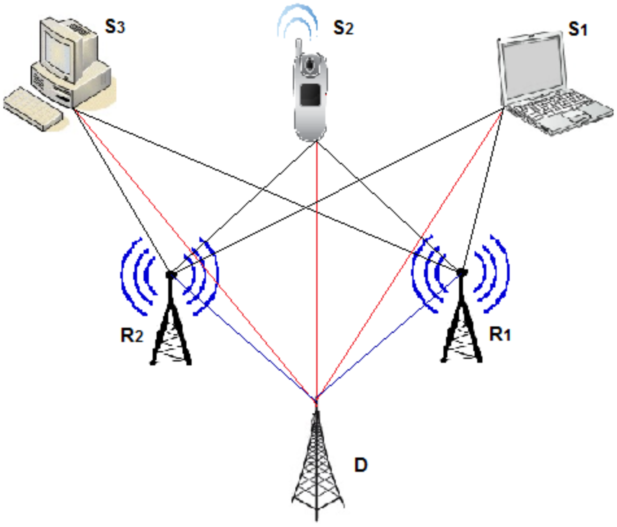

Fig. 1 MUD in multiuser TCMRNs with K=3, M=2. The lines between any two nodes represent the communication links

At the destination, the received signal from the k-th user and from the k-th user to the m-th relay node, respectively, are formulated as follows:

$$y_d^k = \sqrt{E_k} h_{k,d} x_k + n_{k,d} \quad (1)$$
$$y_m^k = \sqrt{E_k} h_{k,m} x_k + n_{k,m} \quad (2)$$

where x_k is the user-k signal, and y_d^k and y_m^k are the k-th user's transmitted signal received at destination and m-th relaying nodes, respectively. $h_{k,d}$ and $h_{k,m}$ are the channel coefficients from the k-th user to the destination and from the k-th user to the m-th relay node, respectively. $n_{k,d}$ and $n_{k,m}$ are the additive white Gaussian noise (AWGN) with the variances being $N_{k,d}$ and $N_{k,m}$, respectively.

For the AF scheme, upon receiving the k-th user's signal, the m-th relay node sends a new signal $x_m^k = \alpha_m^k \cdot y_m^k$ to the destination. The amplification factor α_m^k is given as $\alpha_m^k = 1/\sqrt{E\{|y_m^k|^2\}}$. Through the relay node–destination link, the received signal of the k-th user at the receiver of the base station is:

$$\begin{aligned} y_d^m &= \sqrt{E_m} h_{m,d}^k x_m^k + n_{m,d}^k \\ &= \sqrt{E_m} h_{m,d}^k y_m^k \alpha_m^k + n_{m,d}^k \\ &= \sqrt{\frac{E_k E_m}{E_k |h_{k,m}|^2 + N_{k,m}}} h_{k,m} h_{m,d}^k x_k + \tilde{n}_{m,d}^k \qquad (3) \end{aligned}$$

where $h_{m,d}^k$ is the channel coefficient between the relaying node and the destination link when relaying the source signal, $n_{m,d}^k$ is the AWGN with variance $N_{m,d}^k$, and $\tilde{n}_{m,d}^k$ is the AWGN with variance $\tilde{N}_{m,d}^k$, which is given as $\tilde{N}_{m,d}^k = N_{m,d}^k + \left(E_m |h_{m,d}^k|^2 N_{k,m} / \left(E_m |h_{m,d}^k|^2 + N_{k,m}\right)\right)$.

The received signal-to-noise ratio (SNR) at the receiver terminal of the k-th user after maximum ratio combining (MRC) at the base station can be expressed as [3], [8]:

$$\gamma_k^{AF} = \frac{|h_{k,d}|^2 E_k}{N_{k,d}} + \sum_{m=1}^{M} \frac{\frac{|h_{k,d}|^2 E_k}{N_{k,m}} \cdot \frac{|h_{m,d}^2|^2 E_m}{N_{m,d}^k}}{\frac{|h_{k,m}|^2 E_k}{N_{k,m}} + \frac{|h_{m,d}^2|^2 E_m}{N_{m,d}^k} + 1} \quad (4)$$

where $h_{m,d}$ denote the channel coefficient from the m-th relay to destination, E is the average transmission energy. In this paper we consider the case where each user has identical fading statistics, and assume that all the noise variances are equal, i.e., $N_{k,d} = N_{k,m} = N_{m,d}^k = N_0 = 1/\bar{\gamma}$, where $\bar{\gamma}$ is proportional to all the transmitted SNRs. For more simplification, when the channel is exponentially distributed, we can express the k-th user's received SNR at the destination as:

$$\gamma_k^{AF} = \alpha_k \bar{\gamma} + \sum_m \frac{\beta_{km} C_{km} \bar{\gamma}^2}{\beta_{km} \bar{\gamma} + C_{km} \bar{\gamma} + 1} \quad (5)$$

where:

$$\alpha_k = |h_{k,d}|^2 E_k, \beta_{km} = |h_{k,m}|^2 E_k, C_{km} = |h_{m,d}^k|^2 E_m$$

are the signal powers received at the destination from the k-th user, at the m-th relaying node from the k-th user, and from the intermediate relaying node to the destination, respectively.

III. PERFORMANCE ANALYSIS

In multiuser TCMRNs with MUD, and based on the fact that the CSI is available at the destination from both the relaying nodes and the source, the destination will choose the user who can achieve the largest SNR , i.e., the largest achievable system SNR is $\gamma_s^{AF} = max_k \gamma_k^{AF}$.

Thus, the achievable channel capacity of MUD for AF is given as follows [2]:

$$I_s^{AF} = \frac{1}{2} log_2(1 + \gamma_s^{AF}) = \frac{1}{2} log_2(1 + max_k \gamma_k^{AF}) \quad (6)$$

To analyze the performance of the proposed system, i.e., the outage probability and SEP, we have to provide the cumulative density function (cdf) of γ_s^{AF}.

Based on [2], the cdf of γ_s^{AF} in multiuser TCMRNs with MUD, in large average SNR systems ($\bar{\gamma} \gg 1$), can be approximated as follows:

$$F_{\gamma_s^{AF}}(\gamma) = \frac{1}{2^{KM}} \left(\frac{\gamma}{\bar{\gamma}}\right)^{2K} \prod_{k=1}^{K} \prod_{m=1}^{M} \bar{\alpha}_k (\bar{\beta}_{km} + \bar{C}_{km}) \quad (7)$$

Proof:

For the independent identically distributed RV γ_k^{AF}, the cdf of γ_s^{AF} can be calculated as:

$$\begin{aligned}F_{\gamma_s^{AF}}(\gamma) &= Pr(\gamma_s^{AF} \leq \gamma)\\ &= Pr(max_k \gamma_k^{AF} \leq \gamma)\\ &= \prod_{k=1}^{K} Pr\left[\left(\alpha_k\bar{\gamma} + \sum_{m=1}^{M} \frac{\beta_{km}C_{km}\bar{\gamma}^2}{\beta_{km}\bar{\gamma} + C_{km}\bar{\gamma} + 1}\right) \leq \gamma\right]\\ &= \left(\frac{\gamma}{\bar{\gamma}}\right)^{2K} \prod_{k=1}^{K} \frac{Pr\left[\left(\alpha_k + \sum_{m=1}^{M} \frac{\beta_{km}C_{km}}{\beta_{km} + C_{km} + 1/\bar{\gamma}}\right) \leq \gamma/\bar{\gamma}\right]}{(\gamma/\bar{\gamma})^2}\end{aligned} \tag{8}$$

According to [9, eq. (43)], for an arbitrary user k, it is shown that for independent exponentially RVs α_k, β_{km}, and C_{km} with means of $\bar{\alpha}_k$, $\bar{\beta}_{km}$, and $\bar{C}_{km}$, respectively, for a large achievable SNR system, there is an approximation as follows:

$$\lim_{\bar{\gamma}\to\infty} \frac{Pr\left[\left(\alpha_k + \sum_{m=1}^{M} \frac{\beta_{km}C_{km}}{\beta_{km} + C_{km} + 1/\bar{\gamma}}\right) \leq \gamma/\bar{\gamma}\right]}{(\gamma/\bar{\gamma})^2} = \prod_{m=1}^{M} \frac{\bar{\alpha}_k(\bar{\beta}_{km} + \bar{C}_{km})}{2} \tag{9}$$

Thus, taking (9) into (8) and for a large average SNR $\bar{\gamma}$ system, the asymptotic cdf of γ_s^{AF} for the AF protocol is written as:

$$F_{\gamma_s^{AF}}(\gamma) = \frac{1}{2^{KM}}\left(\frac{\gamma}{\bar{\gamma}}\right)^{2K} \prod_{k=1}^{K}\prod_{m=1}^{M} \bar{\alpha}_k(\bar{\beta}_{km} + \bar{C}_{km}) \tag{10}$$

Next, we will use the cdf of γ_s^{AF} to derive the outage probability and SEP of MUD in TCMRNs.

A. *Outage Probability*

The outage probability of AF-based multiuser TCMRN with MUD, defined as the probability that the channel capacity falls below a specified target transmission rate R is calculated as:

$$\begin{aligned}P_{out}^{AF} &= Pr[I_s^{AF} < R] = Pr[\gamma_s^{AF} < 2^{2R} - 1]\\ &= F_{\gamma_s^{AF}}(2^{2R} - 1)\\ &= \frac{1}{2^{KM}}\left(\frac{2^{2R} - 1}{\bar{\gamma}}\right)^{2K} \prod_{k=1}^{K}\prod_{m=1}^{M} \bar{\alpha}_k(\bar{\beta}_{km} + \bar{C}_{km})\end{aligned} \tag{11}$$

B. *Symbol Error Probability (SEP)*

Generally, the conditional SEP for a specific SNR of the most commonly used signal modulation techniques, including M-ary quadratic-amplitude modulation (M-QAM), M-ary phase-shift keying (M-PSK), and M-ary pulse amplitude modulation (M-PAM), can be written as the uniform expression [10]:

$$P_s(\gamma_s) = AQ\left(\sqrt{B\gamma_s}\right) \tag{12}$$

where A and B are the specific constellation parameters [11] and {Q} is the Gaussian Q function, for certain SNR the average SEP for Rayleigh channel can be expressed as:

$$\begin{aligned}P_s^{AF} = E_{\gamma_s^{AF}}\left\{AQ\left(\sqrt{B\gamma_s^{AF}}\right)\right\} &= AE_{\gamma_s^{AF}}\left\{\left(\sqrt{B\gamma_s^{AF}}\right) < X\right\}\\ &= AE_{\gamma_s^{AF}}\left\{\gamma_s^{AF} < \frac{X^2}{B}\right\}\\ &= AE_X\left\{F_{\gamma_s^{AF}}\left(\frac{X^2}{B}\right)\right\}\end{aligned} \tag{13}$$

where:

X is a normally distributed random variable.

$E_{\gamma_s^{AF}}\{\cdot\}$ is the expectation operator applied on the distribution of γ_s^{AF}.

Using (7), (13) can be asymptotically approximated as:

$$\begin{aligned}P_s^{AF} &\approx A\int_0^\infty \frac{1}{2^{KM}}\left(\frac{x^2}{B\bar{\gamma}}\right)^{2K} \prod_{k=1}^{K}\prod_{m=1}^{M} \bar{\alpha}_k(\bar{\beta}_{km} + \bar{C}_{km})\\ &\qquad \times \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx\\ &= \frac{A}{\sqrt{2\pi}2^{KM}(B\bar{\gamma})^{2K}} \prod_{k=1}^{K}\prod_{m=1}^{M} \bar{\alpha}_k(\bar{\beta}_{km} + \bar{C}_{km})\\ &\qquad \times \int_0^\infty x^{4K} e^{-\frac{x^2}{2}} dx\\ &= \frac{A}{(B\bar{\gamma})^{2K}} \frac{[4K - 1]!}{2^{K(M+2)}[2K - 1]!} \prod_{k=1}^{K}\prod_{m=1}^{M} \bar{\alpha}_k(\bar{\beta}_{km} + \bar{C}_{km})\end{aligned} \tag{14}$$

in which we use [12]

$$\int_0^\infty x^{2n} e^{-px^2} dx = \frac{\sqrt{(\pi/p)}(2n - 1)!}{2^{2n}(n - 1)!\,p^n}$$

IV. RESULTS AND DISCUSSION

We present some results based on the analysis provided in the previous section. Fig. 2 provides the outage probability for one relay with different number of users K = 1, 2, 3 and 4 respectively, as shown in the figure, P_{out} decreases rapidly as the number of users increases. For example, given outage probability of 10^{-3} when we upgrade K from 2 to 3 we will get a performance enhancement of more than 3 dB which confirms the principle of multiuser diversity in cooperative relays.

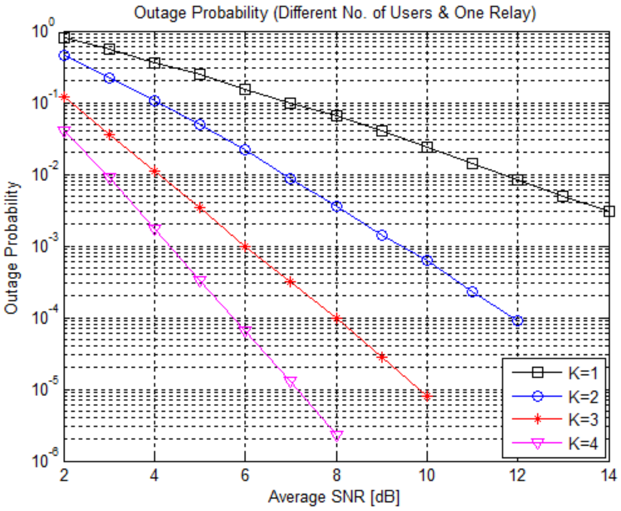

Fig. 2 Outage probability vs. SNR (1 relay)

Fig. 3 shows P_{out} in case of two relays and different number of users. From the figure we can see that the system with two relays provide better performance than the one with one relay.

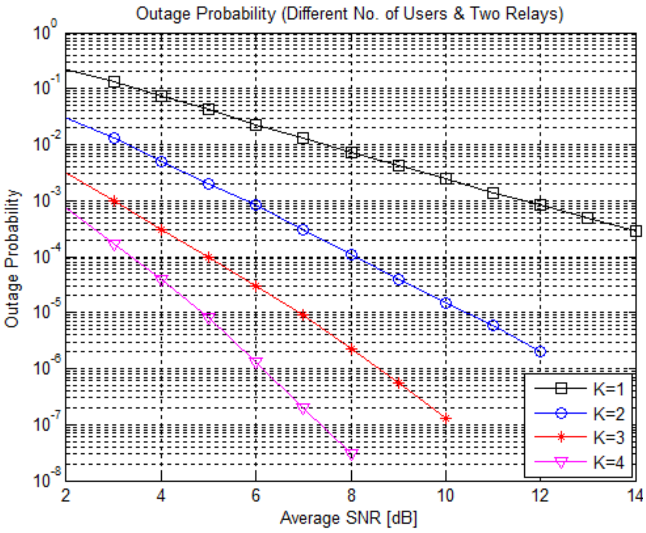

Fig. 3 Outage probability vs. SNR (2 relays)

Fig. 4 and Fig. 5 show the SEP performance of (QPSK) for different users using one relay and two relays respectively. We conclude that in multiuser TCMRNs, the number of users has more effect on the performance of SEP, for example, for one relay (Fig. 4), when the SEP is 10^{-3}, there is an enhancement in the performance of 6 dB when K is 2 over that when K is 1 and there is an enhancement in the performance of 2 dB when k is 3 over that when K is 2. Meanwhile, the performance gain will be reduced when the users increases, that is, as the number of the users increase, this cannot infinitely make a considerable reduction in the SEP.

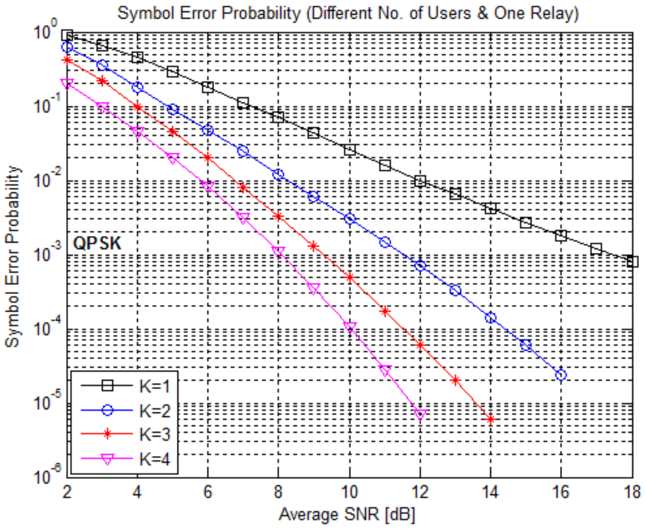

Fig. 4 SEP vs. SNR for one relay

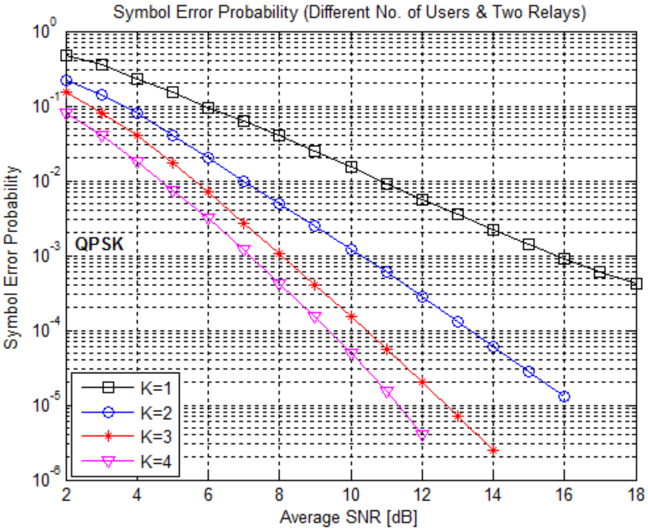

Fig. 5 SEP vs. SNR for two relays

To explain more the effect of the number of relays, Fig. 6 and Fig. 7 show the outage probability and SEP respectively, for one relay and two relays plotted on the same figures for K = 1 and 3. From the two figures we can conclude that the system with two relays provide better performance than the one with one relay. For example, for three users and for outage probability = 10^{-3} when we increase the number of relays from 1 to 2 there would be an enhancement in the performance of more than 3 dB. Also, we can conclude that in multiuser TCMRNs, the relay nodes number has a considerable effect on the performance of SEP, for example, for one user (Fig. 7), when the SEP is 10^{-3}, there would be an enhancement in the performance of more than 1.7 dB for two relays (M = 2) over that of one relay (M = 1), whereas, for three users (K = 3), there would be a performance enhancement of more than 1 dB for two relays (M = 2) over that of one relay (M = 1). Meanwhile, the relative performance gain decreases with the increase of the number of relays, that is, increasing the number of relays cannot infinitely decrease the SEP.

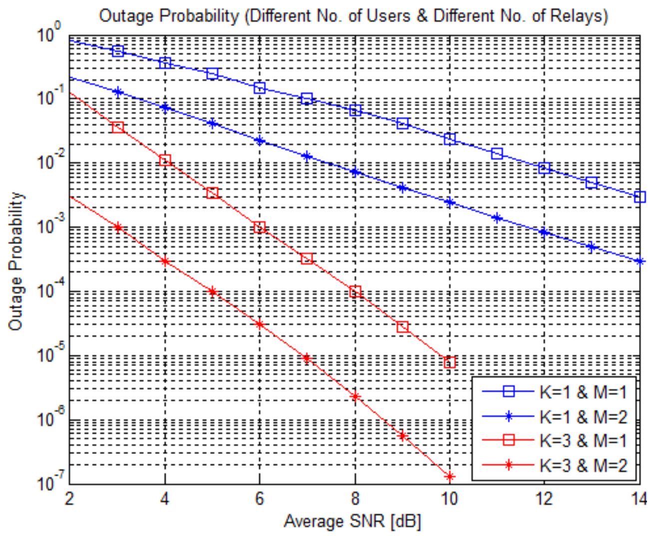

Fig. 6 Outage probability vs. SNR for different number of relays

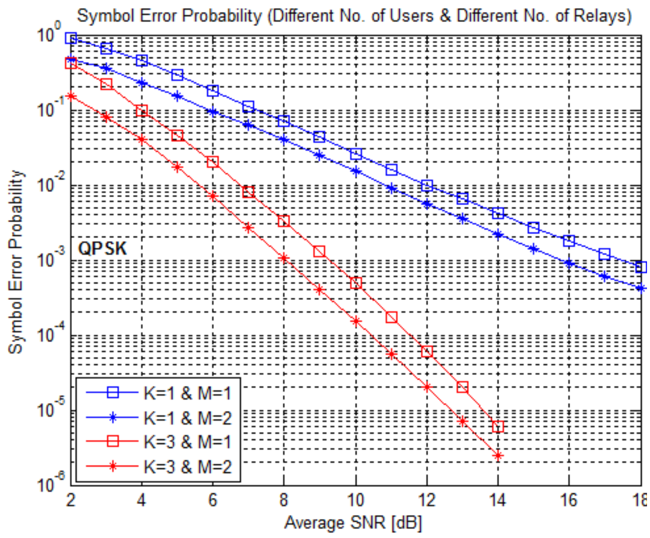

Fig. 7 SEP vs. SNR for different number of relays

V. CONCLUSION

We have derived tight close form formulas of the outage probability and the symbol error probability for MUD in multiuser cooperative multi-relay system. We have *2K* diversity order (*K* number of users) and *M+1* cooperative diversity (*M* available relays and one direct path), these assumptions and derivations are valid for Rayleigh channels and for high SNR. From the results one can see that the system with multi-relays provide better performance than the one with only one relay.

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