Computational Study on Soliton–like Pulses in the Nonlinear RLC Transmission Lines

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Abstract - The nonlinear transmission line is a structure where short-duration pulses called electrical solitons can be created and propagated. In this paper, a method to study soliton–like pulses propagation along the nonlinear RLC transmission lines periodically loaded with voltage dependent capacitances is presented. Based on Kirchhoff’s law taken in the continuum limit, exact soliton solutions of voltage equation are obtained by symbolic computation using an improved tanh–function method. It is shown that the shape of soliton can be controlled well by adjusting the parameters of the line. These solutions may have important applications in communication systems where solitons are used to codify data.

I. INTRODUCTION

Solitons, self-localized waves, are a special class of pulse-shaped waves that propagate without changing their shape in nonlinear dispersive media. A balancing mechanism between nonlinearity and dispersion is responsible for the appearance of solitary wave or pulse which propagates infinitely [1]. The first–reported soliton was a mono–pulse water wave in a narrow canal where the shallow water possessed both nonlinearity and dispersion. The propagation of modulated waves, such as bright or dark solitons, has been the subject of considerable interest for many years. The optical fiber is yet another example of a nonlinear dispersive medium where optical solitons are observed.

In electronics, nonlinear transmission lines (NLTLs) serve as nonlinear dispersive media where electrical solitons can propagate in the form of voltage waves [2]. A NLTL is a ladder network of repeating inductors and capacitors, where the inductors, the capacitors, or both are nonlinear in their response to current and voltage, respectively. The semiconductor devices such as Schottky diodes and heterostructure barrier varactors can also be incorporated into a soliton generating NLTL. These devices are used to generate oscillations with frequencies up to the terahertz range and pulses with subpicosecond durations [3]. Recently, it has been pointed out that a spatially modulated nonlinear dielectric line can be used to generate solitons at high powers and microwave frequencies. In this type of line, the dispersion comes from the periodicity of the line, while the nonlinearity comes from the dielectric material [4].

These lines are of interest because of their applications in several fields. For example, in the linear regime, the NLTLs can be used as phase shifters in phased antenna arrays [5], where time delay can be controlled by means of a DC bias applied to Schottky diodes acting as variable reactance. Under large signal conditions, NLTLs can serve as impulse compressors or frequency multipliers [6]. Recently, these lines have proven to be of great practical use in extremely wideband (frequencies from DC to 100 GHz) focusing and shaping of signals [7], which is usually a hard problem. Very recently, attempts have been made to study analytically and numerically nonlinear excitations using NLTLs, and it has been shown experimentally that solitons can exist in such systems [8].

The main aim of this paper is to present a method to computational study of solitary waves in nonlinear RLC transmission lines. A circuit model of a NLTL with \( N \) identical cells is given in Fig. 1. The series inductance is due to magnetic field effects, and the capacitance is due to electric field coupling between the lines. The losses in the transmission media are depicted by the series and the shunt resistors. These resistors represent the finite conductivity of the conductors and the dielectric insulator between the conductors, respectively. The resistors \( R_1 \) and \( R_2 \), accounting for the transmission line losses, the linear inductance \( L \), and the voltage–dependent capacitance \( C(V) \) are the circuit parameters. The resulting circuit is referred to as a distributed model of a nonlinear RLC transmission lines. A distributed model of the NLTL has been proposed to deal with wide–bandwidth signals.

Interestingly, in constructing proposed lossy nonlinear network, we were motivated with a very interesting example of nanobioelectronics problem. Namely, our group was recently modeled ionic currents along microtubules in the eukaryotic cells [9]. We analytically analyzed the possible use of microtubules as protein structure for building biomolecular nanoscale NLTLs in the context of the polyelectrolyte character of these cytoskeletal filaments [10].

![Figure 1. Schematic representation of one cell of the nonlinear transmission line.](image-url)
II. Model Description and Voltage Equation

We now consider a NLTL constructed by \( N \) elementary cells, where each cell, such as the \( n \)th one, contains a linear inductance \( L \) in the series branch, a nonlinear capacitor \( C(V_b + V_n) \) in the shunt branch with \( V_n \) being the voltage across it and \( V_b \) a constant DC bias voltage, and the linear resistances \( R_1 \) and \( R_2 \), see Fig. 1. The reverse–biased diode or MOS varactor can serve as the nonlinear capacitor, and its differential capacitance is \( C(V_b + V_n) = dq_n/dV_n \) where \( q_n \) denotes the stored charge in the \( n \)th capacitor, and it depends nonlinearly on the voltage \( V_n \). For low voltages around the DC bias voltage \( V_b \), the nonlinear capacitance can be approximated by [11]

\[
C(V_b + V_n) = \frac{dq_n}{dV_n} = C_0 \left( 1 + 2\alpha V_n + 3\beta V_n^2 + \ldots \right) \tag{1}
\]

where \( C_0 \) is the linear capacitance of the capacitor, and \( \alpha \) and \( \beta \) designate the nonlinear coefficients of the capacitor.

Equation (1) can be the second order curve fitting for the diode characteristics or the varactor characteristics according to the sign of nonlinear coefficients \( \alpha \) and \( \beta \). For the diode the sign of coefficient \( \alpha \) is negative and the sign of coefficient \( \beta \) is positive and vice versa for the MOS varactor. In this study, we only consider the case when the perturbation voltage is small in comparison with the equilibrium voltage \( V_b \), therefore, we neglect higher–order terms in (1), keeping only the first two terms of the expansion.

By applying the Kirchhoff current law at node \( n \) whose voltage with respect to the ground is \( v_n \), and applying the Kirchhoff voltage law across the two inductors connected to this node, we obtain:

\[
i_{n-1} - i_n = \frac{dq_n}{dt} \tag{2}
\]

\[
v_n - v_{n+1} = L \frac{di_n}{dt} + R_1i_n. \tag{3}
\]

Similarly, if the voltage across the capacitor is \( V_b + V_n \), where \( V_b \) is the bias voltage of the capacitor, we have:

\[
v_n = R_2(i_{n-1} - i_n) + V_b + V_n. \tag{4}
\]

From (3) we have:

\[
L \frac{di_{n-1}}{dt} = v_{n-1} - v_n - R_1 i_{n-1}. \tag{5}
\]

\[
L \frac{di_n}{dt} = v_n - v_{n+1} - R_1 i_n. \tag{6}
\]

From (2) we have

\[
L \frac{d^2q_n}{dt^2} = L \frac{di_n}{dt} - L \frac{di_n}{dt}, \tag{7}
\]

and including (5) in (6), we get:

\[
L \frac{d^2q_n}{dt^2} = v_{n-1} - 2v_n + v_{n+1} + R_1(i_n - i_{n-1}). \tag{8}
\]

Replacing the expressions for the voltages \( v_{n-1}, v_n \) and \( v_{n+1} \) from (4) to (7), we obtain that voltages of the adjacent nodes on this lossy NLTL are related via partial differential equation (PDE) as follows

\[
L \frac{d^2q_n}{dt^2} = C_0 \left( V_n + \alpha V_n^2 \right) + R_1C_0 \frac{d}{dt} \left( V_n + \alpha V_n^2 \right) - R_2C_0 \frac{d}{dt} \left( V_{n-1} + \alpha V_{n-1}^2 \right) - 2 \frac{d}{dt} \left( V_n + \alpha V_n^2 \right) \tag{9}
\]

\[
\frac{d}{dt} \left( V_{n+1} + \alpha V_{n+1}^2 \right) = V_{n-1} - 2V_n + V_{n+1}. \tag{10}
\]

The right–hand side of (8) can be approximated with partial derivatives with respect to distance \( x \), from the beginning of the line, assuming that the spacing between two adjacent sections is \( \delta \) (i.e., \( x_n = n\delta \)). Let voltage \( V(x, t) \) be a continuous function of the variables \( x \) and \( t \) so that \( V(x, t) = V_0(x, t) \) and \( V_0(x, t) = V(x + \delta, t) \). An approximate continuous PDE can be obtained by using the Taylor expansions

\[
V_{n+1} = V(x + \delta, t) = V(x, t) \pm \delta \frac{\partial V}{\partial x} + \frac{\delta^2 \partial^2 V}{2! \partial x^2} \pm \frac{\delta^3 \partial^3 V}{3! \partial x^3} + \ldots \tag{11}
\]

\[
V_{n-1} - 2V_n + V_{n+1} = \delta^2 \frac{\partial^2 V}{\partial x^2} + \frac{\delta^4 \partial^4 V}{12 \partial x^4}. \tag{12}
\]

By eliminating terms of higher order than \( \delta^4 \) as negligible small and considering that the time variations of local voltage \( V \) are small compared to the constant background voltage \( V_b \), we could safely assume that time derivative is of the order of small parameter \( \varepsilon \) as well as the nonlinear voltage terms \( \alpha V^2 \) are of the order of \( \varepsilon^2 \). Thus, we obtain the following nonlinear PDE for the perturbed voltage \( V \):
\[ R_2 C_0 \delta^2 \frac{\partial^2 V}{\partial t^2} + \delta^2 \frac{\partial^2 V}{\partial \xi^2} - \delta \frac{\partial^2 V}{\partial t \partial \xi} - L C_0 \frac{\partial^2 V}{\partial t^2} + 2a R_1 C_0 \frac{\partial V}{\partial \xi} = 0. \] (11)

III. SOLITON–LIKE PULSES IN NLTL

In this study, the improved tanh–function method (ITM) is applied in order to find the analytical solution of the NLTL [12]. The first step in this method is to introduce the voltage in the form of the travelling wave

\[ V(x,t) = V(\xi); \quad \xi = x - v_0 t, \] (12)

where \( v_0 \) is undetermined parameter that represents the velocity of propagation. A propagating mode solution of (11) can be obtained by substituting (12) into the PDE (11). Then voltage equation (11) becomes the ordinary differential equation (ODE) of the third order

\[ \frac{R_2}{R_1} \delta^2 v_0 \frac{d^2 V}{d\xi^2} + \frac{L}{R_1} (v_0^2 - \delta^2 \omega_c^2) \frac{d^2 V}{d\xi^2} + 2a v_0 \frac{d V}{d\xi} - v_0 \frac{d V}{d\xi} = 0. \] (13)

Here the parameter \( \omega_c \), meaning the cutoff angular frequency, is defined by \( \omega_c = (L C_0)^{1/2} \) and the maximum velocity of the propagation of waves is \( v_{0\text{max}} = \delta \omega_c \). After first integration of ODE (13), we obtain

\[ A \frac{d^2 V}{d\xi^2} + B \frac{d V}{d\xi} - v_0 V + D V^2 = c_1, \] (14)

where \( c_1 \) is an arbitrary constant, whose value depends on the initial conditions. Requiring that all functions \( V(\xi) \), \( dV(\xi) / d\xi \) and \( d^2 V(\xi) / d\xi^2 \) tend to zero as \( \xi \to \infty \), the integration constant \( c_1 \) is set to be zero. The parameters of ODE (14) read as follows:

\[ A = \frac{R_2}{R_1} \delta^2 v_0, \quad B = \frac{L}{R_1} (v_0^2 - \delta^2 \omega_c^2), \quad D = 2a v_0. \] (15)

It is observed form (13) that the sign of the dispersion coefficient, \( A \), is always positive, but the sign of the nonlinearity coefficient, \( D \), depends on the nonlinear characteristics of the capacitance. Therefore, the sign of the product \( AD \) can be positive or negative, which leads to appearance of dark soliton \( (AD < 0) \) or bright soliton \( (AD > 0) \). The parameter \( B \) is always positive and involves the effect of dissipation.

We now introduce the new independent variable \( Y = \tanh(\xi) \) and \( W(Y) = V(\xi) \) [13], as a polynomial series, due to which ODE (14) transforms to the shape

\[ A[1 - Y^2] \frac{d^2 W}{dY^2} + [B(1 - Y^2) - 2a Y(1 - Y^2)] \frac{dW}{dY} - v_0 W + DW^2 = 0. \] (16)

We compare the derivative term of highest–order with the highest order nonlinear term. So, balancing the order of \( W^m \) with the order of \( W^2 \) in (16), we obtain \( m + 2 = 2m \rightarrow m = 2 \). So the solution takes the form

\[ W(Y) = \sum_{i=0}^{m=2} a_i Y^i = a_0 + a_1 Y + a_2 Y^2, \] (17)

where parameters \( a_0, a_1, \) and \( a_2 \) are to be determined. Inserting (17) into (16), we get a system of algebraic equations:

\[
\begin{align*}
y^0 & : 2a_2 A + a_2 B + a_0^2 D - v_0 a_0 = 0, \\
y^1 & : -2a_1 A + 2a_2 B + 2a_0 a_1 D - v_0 a_1 = 0, \\
y^2 & : -8a_2 A - a_1 B + a_0^2 D + 2a_0 a_2 D - v_0 a_2 = 0, \\
y^3 & : 2a_1 A - 2a_2 B + 2a_0 a_2 D = 0, \\
y^4 & : 6a_2 A + a_0^2 D = 0.
\end{align*}
\]

Solving the above system of algebraic equations with the aid of MATLAB, we obtain the following calculated values of the parameters:

\[
a_0 = \frac{1}{D} \left( 6A + \frac{1}{2} v_0 \right); \quad a_1 = \frac{6B}{5D}; \quad a_2 = -\frac{6A}{D} \]

(19)

together with a relation between coefficients \( B \) and \( A \) as \( B^2 = 100A^2 \). Finally, using the determined parameters, the exact soliton solution of the voltage equation (11) reads explicitly

\[ V(x,t) = \frac{1}{D} \left( 6A + \frac{1}{2} v_0 \right) + \frac{6B}{5D} \tanh(x - v_0 t) - \frac{6}{D} \tanh^2(x - v_0 t). \] (20)

It represents a particular combination of a bell–shaped solitary wave, third term of (20), with a kink–like shock–wave, second term. Here, the solitary wave velocity \( v_0 \) is given by the following implicit expression

\[ v_0^2 = 4 \left( 2A^2 + \frac{6}{5} B^2 \right). \] (21)

IV. NUMERICAL SIMULATION AND RESULTS

In order to verify our analytical prediction based on (20), we have also analyzed the NLTL using well–known methods of circuit analysis. To this end, the circuit in Fig. 1 is connected to a source with 50 Ω resistance and a load
with 50 Ω resistance. The number of cells between the source and the load are assumed to be 40. The NLTL is then analyzed using a standard time-domain technique for nonlinear circuits [14]. The method uses finite-difference applied to the differential equations governing the voltages and current of the circuit.

In the simulation, we first consider the NLTL with a varactor, the linear inductance $L = 2.5 \text{nH}$, and the linear resistances $R_1 = 0.1 \text{Ω}$, and $R_2 = 1 \text{Ω}$. The varactor’s parameters are linear capacitance $C_0 = 1 \text{pF}$, and nonlinearity factor $\alpha = 0.2 \text{V}^{-1}$. Because of the positive sign of the product $AD$, as predicted by the analytical solution, a bright soliton will be formed along the transmission line. The NLTL is excited by the input voltage given by [11]:

$$V_m(t) = V_{m\text{sech}} \left( \frac{t - \tau_0}{T_0} \right) \cos(2\pi ft)$$  \hspace{1cm} (22)

where the amplitude $V_m = 1 \text{V}$, the parameters $\tau_0 = 0.3 \text{ ns}$, and $T_0 = 1 \text{ ns}$. The carrier wave frequency is chosen to be $f = 2.5 \text{ GHz}$ in passband of the transmission line, below $\omega_c$.

Propagation of this pulse along the transmission line is simulated and the obtained envelope waveform at different positions, $x$, is depicted in Fig. 2. As seen in this figure, the soliton-like pulse preserves its shape along the propagation path. According to the analytical calculations, this circuit because of its positive nonlinearity cannot support dark solutions. As observed in Fig. 3, it presents a good agreement between the analytical and simulated results at $x = 0.1 \text{ m}$.

Now, we consider the NLTL with a reverse-biased diode and the same values for the inductance and resistances. The diode’s parameters are linear capacitance $C_0 = 1 \text{ pF}$, and nonlinearity factor $\alpha = -0.25 \text{ V}^{-1}$. In this case, the sign of the product $AD$ is negative. Therefore, if this circuit is excited by a dark initial signal, a dark soliton will be generated. The envelope waveform obtained from the finite-difference method at different positions, $x$, is depicted in Fig. 4. As observed in Fig. 5, we get a good agreement between the analytical and simulated results.

According to the analytical calculations, this circuit because of its negative nonlinearity cannot support bright solutions.
improve the propagation of soliton–like pulses, because the amplitude of solitons decreases with the increase of the resistances line.

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